数値計算法

連立一次方程式の数値解法

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連立一次方程式

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1N}x_N = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2N}x_N = b_2 \\ \vdots \\ a_{N1}x_1 + a_{N2}x_2 + a_{N3}x_3 + \cdots + a_{NN}x_N = b_N \end{cases}$$

の解を計算機を用いて数値的に求める

- LU分解法（クラウト法）
- 反復法（ガウス・ザイデル法）

連立一次方程式と行列

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1N}x_N = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2N}x_N = b_2 \\ \vdots \\ a_{N1}x_1 + a_{N2}x_2 + a_{N3}x_3 + \cdots + a_{NN}x_N = b_N \end{cases}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1N} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & a_{N3} & \cdots & a_{NN} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{pmatrix}$$

連立一次方程式と逆行列

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{pmatrix} \quad \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} \quad \vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{pmatrix}$$

$$A\vec{x} = \vec{b} \Rightarrow \vec{x} = A^{-1}\vec{b}$$

$$A\vec{u}_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad A\vec{u}_2 = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix} \quad \ldots \quad A\vec{u}_N = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

$$\Rightarrow A^{-1} = (\vec{u}_1, \vec{u}_2, \cdots, \vec{u}_N)$$

三角行列で表現できる連立方程式

$$\begin{cases} a_{11}x_1 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \\ \vdots \\ a_{N1}x_1 + a_{N2}x_2 + a_{N3}x_3 + \cdots + a_{NN}x_N = b_N \end{cases}$$

$$\begin{cases} x_1 = \dfrac{b_1}{a_{11}} \\ x_2 = \dfrac{b_2 - a_{21}x_1}{a_{22}} \\ \vdots \\ x_N = \dfrac{b_N - a_{N1}x_1 - a_{N2}x_2 - a_{N3}x_3 - \cdots - a_{NN-1}x_{N-1}}{a_{NN}} \end{cases}$$

三角行列の連立方程式(左下)

$$\begin{pmatrix} a_{0\ 0} & 0 & \cdots & 0 \\ a_{1\ 0} & a_{1\ 1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{N-1\ 0} & a_{N-1\ 1} & \cdots & a_{N-1\ N-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{pmatrix} = \begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_{N-1} \end{pmatrix}$$

$$\begin{cases} x_0 = \dfrac{b_0}{a_{00}} \\ x_k = \dfrac{1}{a_{kk}} \left(b_k - \displaystyle\sum_{m=0}^{k-1} a_{km}x_m \right) \end{cases}$$

$$k = 1, 2, 3, \cdots, N-1$$

三角行列の連立方程式(左下)

$$x_0 = \frac{b_0}{a_{00}} \quad x_k = \frac{1}{a_{kk}}\left(b_k - \sum_{m=0}^{k-1} a_{km}x_m\right) \quad k = 1, 2, 3, \cdots, N-1$$

```
double s, a[N][N], b[N], x[N];
int k, m;
x[0] = b[0]/a[0][0];
for (k=1; k<N; k++) {
  s = b[k];
  for (m=0; m<k; m++) {
    s = s - a[k][m]*x[m];
  }
  x[k] = s/a[k][k];
}
```

三角行列の連立方程式(右上)

$$\begin{pmatrix} a_{0\ 0} & a_{0\ 1} & \cdots & a_{0\ N-1} \\ 0 & a_{1\ 1} & \cdots & a_{1\ N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{N-1\ N-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{pmatrix} = \begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_{N-1} \end{pmatrix}$$

$$\begin{cases} x_{N-1} = \dfrac{b_{N-1}}{a_{N-1\ N-1}} \\ x_k = \dfrac{1}{a_{kk}}\left(b_k - \displaystyle\sum_{m=k+1}^{N-1} a_{km}x_m\right) \end{cases}$$

$$k = N-2, N-3, N-4, \cdots, 0$$

三角行列の連立方程式(右上)

$$x_{N-1} = \frac{b_{N-1}}{a_{N-1\ N-1}},\ x_k = \frac{1}{a_{kk}}\left(b_k - \sum_{m=k+1}^{N-1} a_{km}x_m\right)$$

$$k = N-2, N-3, N-4, \cdots, 0$$

```
x[N-1] = b[N-1]/a[N-1][N-1];
for (k=N-2; k>=0; k--) {
  s = b[k];
  for (m=k+1; m<N; m++) {
    s = s - a[k][m]*x[m];
  }
  x[k] = s/a[k][k];
}
```

LU分解

- 一般に正方行列は左下三角行列と右上三角行列の積で表すことができる．

$$\overset{A}{\begin{pmatrix} A_{0\ 0} & A_{0\ 1} & \cdots & A_{0\ N-1} \\ A_{1\ 0} & A_{1\ 1} & \cdots & A_{1\ N-1} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N-1\ 0} & A_{N-1\ 1} & \cdots & A_{N-1\ N-1} \end{pmatrix}}$$

$$= \underset{L}{\begin{pmatrix} L_{0\ 0} & 0 & \cdots & 0 \\ L_{1\ 0} & L_{1\ 1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ L_{N-1\ 0} & L_{N-1\ 1} & \cdots & L_{N-1\ N-1} \end{pmatrix}} \underset{U}{\begin{pmatrix} 1 & U_{0\ 1} & \cdots & U_{0\ N-1} \\ 0 & 1 & \cdots & U_{1\ N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}}$$

$$A = LU$$

LU分解のアルゴリズム

- クラウト法

$$(0 \le k \le N-1)$$

$$\begin{cases} U_{kk} = 1 \\ L_{ik} = \begin{cases} 0 & (0 \le i < k) \\ A_{ik} - \displaystyle\sum_{m=0}^{k-1} L_{im}U_{mk} & (k \le i \le N-1) \end{cases} \\ U_{ki} = \begin{cases} 0 & (0 \le i < k) \\ \dfrac{1}{L_{kk}}\left(A_{ki} - \displaystyle\sum_{m=0}^{k-1} L_{km}U_{mi}\right) & (k+1 \le i \le N-1) \end{cases} \end{cases}$$

LU分解法（クラウト法）

$A\boldsymbol{x}=\boldsymbol{b}$を解く

AをLU分解する $A=LU \Rightarrow LU\boldsymbol{x}=\boldsymbol{b}$

⇩

$\boldsymbol{y}=U\boldsymbol{x}$とおいて$L\boldsymbol{y}=\boldsymbol{b}$を解き$\boldsymbol{y}=L^{-1}\boldsymbol{b}$を求める

⇩

$U\boldsymbol{x}=\boldsymbol{y}$を解き$\boldsymbol{x}=U^{-1}\boldsymbol{y}=U^{-1}L^{-1}\boldsymbol{b}$を求める

例題1

$$\begin{pmatrix} 2 & -1 & 10 \\ -1 & 1 & 5 \\ 4 & -3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 20 \\ 14 \\ -6 \end{pmatrix}$$ を解く

正しい答は $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 2 \end{pmatrix}$

LU分解法のプログラム

```
int main(void) {
  int i, j;
  double a[N][N] = {{2.0, -1.0, 10.0},
                    {-1.0, 1.0, 5.0},
                    {4.0, -3.0, 1.0}};
  double b[N] = {20.0, 14.0, -6.0};
  double l[N][N], u[N][N], x[N], y[N];
  show(a, b);
  ludec(a, l, u);
  solvl(l, b, y);
  show(l, y);
  solvu(u, y, x);
  show(u, x);
  return 0;
}
```

LU分解法のプログラム

```
#include <stdio.h>
#define N 3
void ludec(double A[N][N], double L[N][N], double U[N][N]);
void solvl(double L[N][N], double B[N], double Y[N]);
void solvu(double U[N][N], double Y[N], double X[N]);
void show(double M[N][N], double V[N]);
```

```
void ludec(double A[N][N], double L[N][N], double U[N][N]) {
  int i, k, m;
  for (i=0; i<N; i++) {
    L[i][0] = A[i][0];
    U[0][i] = A[0][i]/L[0][0];
  }
  for (k=1; k<N; k++) {
    for (i=0; i<k; i++) {
      L[i][k] = 0.0;
      U[k][i] = 0.0;
    }
    for (i=k; i<N; i++) {
      L[i][k] = A[i][k];
      for (m=0; m<k; m++) {L[i][k] = L[i][k] - L[i][m]*U[m][k];}
    }
    U[k][k] = 1.0;
    for (i=k+1; i<N; i++) {
      U[k][i] = A[k][i];
      for (m=0; m<k; m++) {U[k][i] = U[k][i] - L[k][m]*U[m][i];}
      U[k][i] = U[k][i]/L[k][k];
    }
  }
}
```

$$\begin{cases} U_{kk} = 1 \\ L_{ik} = \begin{cases} 0 & (0 \le i < k) \\ A_{ik} - \sum_{m=0}^{k-1} L_{im} U_{mk} & (k \le i \le N-1) \end{cases} \\ U_{ki} = \begin{cases} 0 & (0 \le i < k) \\ \frac{1}{L_{kk}} \left(A_{ki} - \sum_{m=0}^{k-1} L_{km} U_{mi} \right) & (k+1 \le i \le N-1) \end{cases} \end{cases}$$

/* *LU*分解のための関数 */

左下三角行列

```
void solvl(double a[N][N], double b[N], double x[N])
{
  int k, m;
  double s;
  x[0] = b[0]/a[0][0];
  for (k=1; k<N; k++) {
    s = b[k];
    for (m=0; m<k; m++) {
      s = s - a[k][m]*x[m];
    }
    x[k] = s/a[k][k];
  }
}
```

右上三角行列

```
void solvu(double a[N][N], double b[N], double x[N])
{
  int k, m;
  double s;
  x[N-1] = b[N-1]/a[N-1][N-1];
  for (k=N-2; k>=0; k--) {
    s = b[k];
    for (m=k+1; m<N; m++) {
      s = s - a[k][m]*x[m];
    }
    x[k] = s/a[k][k];
  }
}
```

行列とベクトルの出力

```
void show(double m[N][N], double v[N]) {
  int i, j;
  printf("------\n");
  for (i=0; i<N; i++) {
    for (j=0; j<N; j++) {
      printf("%f ", m[i][j]);
    }
    printf("| %f\n", v[i]);
  }
}
```

例題1（計算結果）

```
          ------
 2.00000 -1.00000 10.00000 | 20.00000
-1.00000  1.00000  5.00000 | 14.00000
 4.00000 -3.00000  1.00000 | -6.00000
          ------
 2.00000  0.00000  0.00000 | 10.00000
-1.00000  0.50000  0.00000 | 48.00000
 4.00000 -1.00000  1.00000 |  2.00000
          ------
 1.00000 -0.50000  5.00000 |  4.00000
 0.00000  1.00000 20.00000 |  8.00000
 0.00000  0.00000  1.00000 |  2.00000
```

直接法と反復法

- 直接法
  - ガウスの消去法
  - ガウス・ジョルダン法
  - LU分解法

  $\rightarrow$ Nが大きくなると計算が困難
- 反復法
  - ヤコビの方法
  - ガウス・ザイデル法

反復法

$$A\vec{x} = \vec{b}$$
$$A \equiv M - N$$
$$(M-N)\vec{x} = \vec{b}$$
$$M\vec{x} = \vec{b} + N\vec{x}$$
$$\vec{x} = M^{-1}\vec{b} + M^{-1}N\vec{x}$$
$$\vec{x}_0 \equiv M^{-1}\vec{b}$$
$$V \equiv M^{-1}N$$
$$\vec{x} = \vec{x}_0 + V\vec{x}$$

$$\vec{x} = \vec{x}_0 + V\vec{x}$$
$$\vec{x}_1 = \vec{x}_0 + V\vec{x}_0$$
$$\vec{x}_2 = \vec{x}_0 + V\vec{x}_1$$
$$= \vec{x}_0 + V(\vec{x}_0 + V\vec{x}_0)$$
$$= \vec{x}_0 + V\vec{x}_0 + V^2\vec{x}_0$$
$$\vec{x}_n = \vec{x}_0 + V\vec{x}_{n-1}$$
$$= \vec{x}_0 + V\vec{x}_0 + V^2\vec{x}_0 + \cdots + V^n\vec{x}_0$$

反復法の終了条件

$$\vec{x} = \vec{x}_0 + \vec{F}(\vec{x})$$
$$\vec{x}_{n+1} = \vec{x}_0 + \vec{F}(\vec{x}_n)$$
$$\lim_{n\to\infty} \vec{x}_n = \vec{x}$$
$$\Delta \equiv |\vec{x}_n - \vec{x}_{n-1}|$$
$$\vec{x}_n = \left(x_0^{(n)}, x_1^{(n)}, x_2^{(n)}, \cdots, x_{N-1}^{(n)}\right)$$
$$\Delta = \sqrt{\sum_{i=0}^{N-1} \left(x_i^{(n)} - x_i^{(n-1)}\right)^2} < \delta$$

ヤコビの方法

$$A\vec{x} = \vec{b} \qquad A \equiv M - N \qquad \vec{x} = M^{-1}\vec{b} + M^{-1}N\vec{x}$$

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{NN} \end{pmatrix} - \begin{pmatrix} 0 & -a_{11} & \cdots & -a_{1N} \\ -a_{21} & 0 & \cdots & -a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{N1} & -a_{N2} & \cdots & 0 \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{NN} \end{pmatrix}^{-1} = \begin{pmatrix} 1/a_{11} & 0 & \cdots & 0 \\ 0 & 1/a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1/a_{NN} \end{pmatrix}$$

ガウス・ザイデル法

$$\begin{pmatrix} a_{11} & a_{11} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{pmatrix} - \begin{pmatrix} 0 & -a_{11} & \cdots & -a_{1N} \\ 0 & 0 & \cdots & -a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}$$

$$x_i^{(n+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(n+1)} - \sum_{j=i+1}^{N} a_{ij} x_j^{(n)} \right)$$

ガウス・ザイデル法

$$\sum_{j=0}^{N-1} a_{ij} x_j = b_i$$

$$\sum_{j=0}^{i-1} a_{ij} x_j + a_{ii} x_i + \sum_{j=i+1}^{N-1} a_{ij} x_j = b_i$$

$$x_i = \frac{1}{a_{ii}} \left(b_i - \sum_{j=0}^{i-1} a_{ij} x_j - \sum_{j=i+1}^{N-1} a_{ij} x_j \right)$$

$$x_i^{(n+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=0}^{i-1} a_{ij} x_j^{(n+1)} - \sum_{j=i+1}^{N-1} a_{ij} x_j^{(n)} \right)$$

例題2

$$\begin{pmatrix} 3 & 1 & 1 \\ 1 & 5 & 2 \\ 1 & 2 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 21 \\ 30 \end{pmatrix}$$ を解く

正しい答は $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$

ガウス・ザイデル法の関数

```
double gz(double a[N][N], double b[N], double x[N]){
  double xi, err = 0.0;
  int i, j;
  for (i=0; i<N; i++) {
    xi = b[i];
    for (j=0; j<N; j++) {
      if (i!=j) xi = xi - a[i][j]*x[j];
    }
    xi = xi/a[i][i];
    err = err + (xi - x[i])*(xi - x[i]);
    x[i] = xi;
  }
  return sqrt(err);
}
```

ガウス・ザイデル法のプログラム

```
#include <stdio.h>
#include <math.h>
#define N 3
#define NN 100
#define DELTA 1.0e-9
double gz(double a[N][N], double b[N], double x[N]);
void showv(int i, double x[N], double err);
```

ガウス・ザイデル法のプログラム

```
int main(void) {
  double a[N][N] = {{3.0, 1.0, 1.0},
                    {1.0, 5.0, 2.0},
                    {1.0, 2.0, 5.0}};
  double b[N] = {10.0, 21.0, 30.0};
  double x[N] = {1.0, 1.0, 1.0};
  int i;
  double err;
  for (i=0; i<NN; i++) {
    err = gz(a, b, x);
    showv(i, x, err);
    if (err < DELTA) return 0;
  }
  printf("NOT CONVERGENT\n");
  return 1;
}
```

ガウス・ザイデル法のプログラム

```
void showv(int i, double x[N], double
err){
  int j;
  printf("--- %d ---\n", i);
  for (j=0; j<N; j++) {
    printf("x[%d] = %f\n", j, x[j]);
  }
  printf("ERROR = %f\n", err);
}
```

例題2（計算結果）

```
--- 0 ---
x[0] = 2.666667
x[1] = 3.266667
x[2] = 4.160000
ERROR = 4.230976
--- 1 ---
x[0] = 0.857778
x[1] = 2.364444
x[2] = 4.882667
ERROR = 2.146702
--- 2 ---
x[0] = 0.917630
x[1] = 2.063407
x[2] = 4.991111
ERROR = 0.325524
--- 3 ---
x[0] = 0.981827
x[1] = 2.007190
x[2] = 5.000759
ERROR = 0.085877
--- 4 ---
x[0] = 0.997350
x[1] = 2.000227
x[2] = 5.000439
ERROR = 0.017017
--- 5 ---
x[0] = 0.999778
x[1] = 1.999869
x[2] = 5.000097
ERROR = 0.002478
.
.
--- 10 ---
x[0] = 1.000000
x[1] = 2.000000
x[2] = 5.000000
ERROR = 0.000000
--- 11 ---
.
.
--- 14 ---
x[0] = 1.000000
x[1] = 2.000000
x[2] = 5.000000
ERROR = 0.000000
```

例題3

$$\begin{pmatrix} 2 & -1 & 10 \\ -1 & 1 & 5 \\ 4 & -3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 20 \\ 14 \\ -6 \end{pmatrix}$$

をガウス・ザイデル法で解く

正しい答は $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 2 \end{pmatrix}$

例題3（計算結果）

```
--- 0 ---
x[0] = 5.500e+00
x[1] = 1.450e+01
x[2] = 1.550e+01
ERROR = 2.032e+01
--- 1 ---
x[0] = -6.025e+01
x[1] = -1.238e+02
x[2] = -1.362e+02
ERROR = 2.156e+02
--- 2 ---
x[0] = 6.294e+02
x[1] = 1.325e+03
x[2] = 1.450e+03
ERROR = 2.256e+03
.
.
.
.
--- 98 ---
x[0] = 4.564e+100
x[1] = 9.586e+100
x[2] = 1.050e+101
ERROR = 1.636e+101
--- 99 ---
x[0] = -4.772e+101
x[1] = -1.002e+102
x[2] = -1.098e+102
ERROR = 1.711e+102
NOT CONVERGENT
```

例題4

$$\begin{pmatrix} 2 & -1 & 10 \\ -1 & 1 & 5 \\ 4 & -3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 20 \\ 14 \\ -6 \end{pmatrix}$$ を

$$\begin{pmatrix} 2 & -1 & 10 \\ 4 & -3 & 1 \\ -1 & 1 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 20 \\ -6 \\ 14 \end{pmatrix}$$

としてからガウス・ザイデル法で解く

正しい答は $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 2 \end{pmatrix}$

例題4（計算結果）

```
--- 0 ---
x[0] = 5.500e+00
x[1] = 9.667e+00
x[2] = 1.967e+00
ERROR = 9.81e+00
--- 1 ---
x[0] = 5.000e+00
x[1] = 9.322e+00
x[2] = 1.936e+00
ERROR = 6.08e-01
--- 2 ---
x[0] = 4.983e+00
x[1] = 9.290e+00
x[2] = 1.939e+00
ERROR = 3.67e-02
.
.
--- 100 ---
x[0] = 4.040e+00
x[1] = 8.052e+00
x[2] = 1.998e+00
ERROR = 2.18e-03
--- 101 ---
x[0] = 4.038e+00
x[1] = 8.050e+00
x[2] = 1.998e+00
ERROR = 2.11e-03
--- 102 ---
.
.
.
.
--- 242 ---
x[0] = 4.000e+00
x[1] = 8.000e+00
x[2] = 2.000e+00
ERROR = 2.08e-05
--- 243 ---
.
.
--- 546 ---
x[0] = 4.000e+00
x[1] = 8.000e+00
x[2] = 2.000e+00
ERROR = 9.86e-10
```