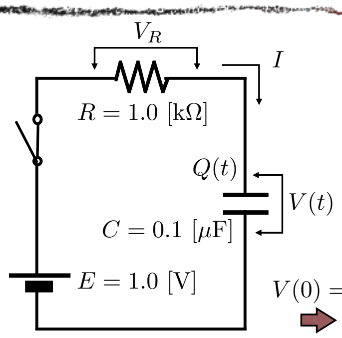
数値計算と次元

RC直列回路の過渡現象



$$V_R = IR = R\frac{d}{dt}Q(t)$$
$$Q(t) = CV(t)$$
$$V_R + V(t) = E$$
$$RC\frac{d}{dt}V(t) + V(t) = E$$

$V(0)=0$

⇨ $V(t) = E\left(1+\exp\left[-\frac{t}{RC}\right]\right)$

$RC = 1.0$ [kΩ] $\times$ 0.1 [μF]$=$ 0.1[ms]

数式と単位系

$$\frac{d}{dt}V(t) = -\frac{1}{RC}V(t) + \frac{E}{RC}$$

$E = 1.0$ [V]$=$ 0.001 [kV]

$RC = \tau = 0.1$ [ms]$=$ 100 [μs]

- 電圧を[V]，時間を[ms]で表すと
  $$\frac{d}{dt}V(t) = -10V(t) + 10$$
- 電圧を[kV]，時間を[μs]で表すと
  $$\frac{d}{dt}V(t) = -0.01V(t) + 0.00001$$

➪単位によって数式が変わってしまう

変数の無次元化

$$\frac{d}{dt}V(t) = -\frac{1}{RC}V(t) + \frac{E}{RC}$$

$E = 1.0$ [V]

$RC = \tau = 0.1$ [ms]

- 電圧をE，時間をτで測ることにする

$$x = \frac{t}{RC} = \frac{t}{\tau}, \quad y = \frac{V(\tau x)}{E} \quad \Rightarrow \quad \frac{d}{dx}y(x) = -y(x) + 1$$

➪無次元な量だけの方程式になる

問題の無次元化

次元を持った問題

$$\frac{d}{dt}V(t) = -\frac{1}{RC}V(t) + \frac{E}{RC}$$

⇩

無次元化

$\tau = RC \quad x = \frac{t}{\tau} \quad y(x) = \frac{V(\tau x)}{E}$

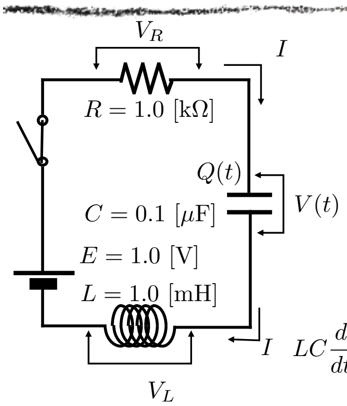
$$\frac{d}{dx}y(x) = -y(x) + 1$$

⇩

適切な単位系を選び次元を復活

$$V(t) = E \cdot y(t/\tau)$$

LCR直列回路の過渡現象



$$V_R = IR = R\frac{d}{dt}Q(t)$$
$$V_L = L\frac{dI}{dt} = L\frac{d^2}{dt^2}Q(t)$$
$$Q(t) = CV(t)$$
$$V_L + V_R + V(t) = E$$
$$LC\frac{d^2}{dt^2}V(t) + RC\frac{d}{dt}V(t) + V(t) = E$$

LCR直列回路の過渡現象

$$LC\frac{d^2}{dt^2}V(t) + RC\frac{d}{dt}V(t) + V(t) = E$$

$$L = 1.0\ [\mathrm{mH}] \qquad C = 0.1\ [\mu\mathrm{F}] \qquad R = 1.0\ [\mathrm{k}\Omega]$$

$$\begin{aligned} LC &= 1.0\ [\mathrm{mH}] \times 0.1\ [\mu\mathrm{F}] & RC &= 1.0\ [\mathrm{k}\Omega] \times 0.1\ [\mu\mathrm{F}] \\ &= 1.0 \times 10^{-10}\ [\mathrm{s}^2] & &= 1.0 \times 10^{-4}\ [\mathrm{s}] \\ &\equiv \tau_1^2 & &\equiv \tau_2 \\ \tau_1 &= 1.0 \times 10^{-5}[\mathrm{s}] & \tau_2 &= 0.1[\mathrm{ms}] \\ &= 10\ [\mu\mathrm{s}] & & \end{aligned}$$

$$\tau_1^2\frac{d^2V}{dt^2} + \tau_2\frac{dV}{dt} + V = E$$

LCR直列回路の過渡現象

$$x \equiv \frac{t}{\tau_1} \quad y \equiv \frac{V}{E} \quad 2a \equiv \frac{\tau_2}{\tau_1}$$

$$\tau_1^2\frac{d^2V}{dt^2} + \tau_2\frac{dV}{dt} + V = E$$

$$\frac{d^2y}{dx^2} + 2a\frac{dy}{dx} + y = 1$$

$$a = \frac{\tau_2}{2\tau_1} = \frac{0.1\ [\mathrm{ms}]}{2 \times 10\ [\mu\mathrm{s}]} = 5$$

定係数2階常微分方程式の解

$$\frac{d^2y}{dx^2} + 2a\frac{dy}{dx} + y = 1 \qquad y(0) = \left.\frac{dy}{dx}\right|_{x=0} = 0$$

1) $a^2 > 1 \ \rightarrow b \equiv \sqrt{a^2 - 1}$

$$y(x) = 1 - e^{-ax}\left[\cosh(bx) + \frac{a}{b}\sinh(bx)\right]$$

2) $a^2 = 1$

$$y(x) = 1 - e^{-x}(1 + x)$$

3) $a^2 < 1 \ \rightarrow b \equiv \sqrt{1 - a^2}$

$$y(x) = 1 - e^{-ax}\left[\cos(bx) + \frac{a}{b}\sin(bx)\right]$$